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Crossover of quantum Loschmidt echo from golden-rule decay to perturbation-independent decay

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We study the crossover of the quantum Loschmidt echo (or fidelity) from the golden-rule regime to the perturbation-independent exponential decay regime by using the kicked top model. It is shown that the deviation of the perturbation-independent decay of the averaged fidelity from the Lyapunov decay results from quantum fluctuations in individual fidelity, which are caused by the coherence in the initial coherent states. With an averaging procedure suppressing the quantum fluctuations effectively, the perturbation-independent decay is found to be close to the Lyapunov decay. We also show that the Fourier transform of the fidelity is determined directly by the initial state and the eigenstates of the Floquet operators of the two classically chaotic systems concerned. The absolute value part and the phase part of the Fourier transform of the fidelity are found to be divided into several correlated parts, which is a manifestation of the coherence of the initial coherent state. In the whole crossover region, some important properties of the fidelity, such as the exponent of its exponential decay and the short initial time within which the fidelity almost does not change, are found to be closely related to the properties of the central part of its Fourier transform.

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I. INTRODUCTION

The quantum Loschmidt echo $M(t)$ measures the overlap of the evolution of the same initial state under two slightly different Hamiltonians in the classical limit,

$$M(t)=|\langle\Phi_0|\exp(iHt)\exp(-iH_0t)|\Phi_0\rangle|^2. \qquad (1)$$

Here H_0 is the Hamiltonian of a classically chaotic system and

$$H=H_0+\kappa V, \qquad (2)$$

with κ being a small quantity.

This quantity characterizes the stability of quantum motion under a small change of the Hamiltonian, e.g., by the interaction with the environment, it is thus called "fidelity" and is of great interest in the fast developing field of quantum information [1,2].

Moreover, a relationship between this quantity and the Lyapunov exponent that characterizes the classical chaos has been established analytically by using the semiclassical theory by Jalabert and Pastawski [3], which has been confirmed numerically in several models [4–9]. Therefore, this quantity has attracted a great attention from the community of quantum chaos [10–13].

It has been shown that several regimes exist. In the golden-rule regime of the perturbation parameter, above a perturbative border, the fidelity has been found to have a simple decaying behavior, $M(t)\propto\exp(-\Gamma_L t)$ [4,6,14], where Γ_L is the half-width of the local density of states (LDOS) ρ_L, which has a Lorentzian form (Breit-Wigner form),

$$\rho_L(E)=\frac{\Gamma_L/2\pi}{E^2+\Gamma_L^2/4}, \qquad (3)$$

with $\Gamma_L=2\pi\rho U^2$. Here ρ is the density of states and U is the typical transition matrix element of κV between the eigenstates of H_0. As the perturbation parameter κ is increased, the exponential decay of $M(t)$ deviates from $\exp(-\Gamma_L t)$, when Γ_L becomes comparable to the bandwidth of H_0, and ρ_L deviates from the Lorentzian form notably. A decay with the Lyapunov exponent λ of the underlying classical chaotic dynamics, $M(t)\propto\exp(-\lambda t)$, can appear in this regime of the perturbation parameter when the (effective) Planck constant is small enough and the initial state is chosen suitably, e.g., a narrow wave packet [3]. An interesting feature of this decay of $M(t)$ is that it is perturbation independent, in the sense that it is irrelevant to the strength of the perturbation κV, and is determined by the classical behavior of the system H_0 (and also that of H due to the smallness of the perturbation in the classical limit). Based on the semiclassical theory and the random matrix theory analysis, a transition of $M(t)$ from the Γ_L decay to the λ decay has been conjectured to occur at $\Gamma_L=\lambda$ [4]. Numerical results in several models support the conjecture [4,7,9].

The validity of results of the semiclassical theory depends on the value of the (effective) Planck constant. When the (effective) Planck constant is not small enough, the validity should be checked by direct quantum mechanical calculations. In fact, the behavior of the fidelity in this case is still not quite clear. For example, whether there is a sharp transition from the Γ_L decay to the λ decay, or the transition occurs in a finite regime of the perturbation parameter. It is even unclear whether $M(t)$ could decay with the Lyapunov exponent. Indeed, a perturbation-independent, but slower than $\exp(-\lambda t)$, decay has been observed in the kicked top model [4], the mechanism of which is still not clear. Meanwhile, the random matrix theory seems unsuitable for such problems, since perturbation-independent decay of $M(t)$ is usually initial-state dependent, the feature of which is hard to be captured by the random matrix theory treatment.

The quantity used in this paper in analyzing properties of the fidelity $M(t)$ is its Fourier transform, denoted by $F_M(E)$ in what follows. As will be shown in Sec. II that $F_M(E)$ is determined directly by properties of the initial state and of the (quasi)energy eigenstates of the two classically chaotic systems concerned. The role played by $F_M(E)$ in understanding the behavior of $M(t)$ is similar to that by the LDOS for the survival probability $P(t)=|\langle\alpha|\exp(-iHt)|\alpha\rangle|^2$.

In this paper, properties of $F_M(E)$ and their relations to those of $M(t)$ will be investigated numerically in the kicked top model [15]. The fidelity in this model has been studied in Refs. [4,14,13], and here we will concentrate on the issues unaddressed in the previous work. Specifically, in addition to properties of $M(t)$ related to the function $F_M(E)$, we focus ourself on the following problems:

(i) The mechanism of the perturbation-independent decay slower than $\exp(-\lambda t)$.

(ii) Due to the small difference between H_0 and H in the classical limit, the classical counterpart of the fidelity $M(t)$ should change quite slowly when t is small enough (see Ref. [9] for numerical results in another model). In the quantum mechanical case, a similar phenomenon should be of practical interest in quantum computing. However, it is not clear whether or not the phenomenon exists.

The paper is organized as the follows. The function F_M is introduced in Sec. II. In Sec. III, we shall present our numerical investigation of the problems mentioned above, in particular, for properties of the amplitude and the phase of the function $F_M(E)$ with initial coherent states. The perturbation-independent decay slower than $\exp(-\lambda t)$ will be shown to be due to large quantum fluctuations in individual fidelity, which are caused by the coherence possessed by the initial coherent states, when the (effective) Planck constant is not small enough. It will be shown that the fidelity $M(t)$ does not decay obviously within a short initial time, the length of which is determined by some properties of the function $F_M(E)$. The relationship between the decaying rate of $M(t)$ after the short initial time and some properties of the function $F_M(E)$ will also be shown numerically. Conclusions and discussions will be given in Sec. IV.

II. FUNCTION $F_M(E)$ AS THE FOURIER TRANSFORM OF $M(t)$

In this paper, the eigenstates of the two Hamiltonians H_0 and H are denoted by $|\alpha\rangle$ and $|\beta\rangle$, respectively, with eigenenergies E_α and E_β,

$$H_0|\alpha\rangle=E_\alpha|\alpha\rangle,\quad H|\beta\rangle=E_\beta|\beta\rangle. \tag{4}$$

The expanding coefficient of $|\beta\rangle$ in $|\alpha\rangle$ is indicated by $C_{\beta\alpha}$, $C_{\beta\alpha}=\langle\alpha|\beta\rangle$. The LDOS $\rho_L(E)$ of an eigenstate $|\alpha\rangle$ of H_0 is determined directly by the eigensolutions of the two Hamiltonians concerned,

$$\rho_L(E)=\sum_\beta |C_{\beta\alpha}|^2\delta[E-(E_\beta-E_\alpha)]. \tag{5}$$

As is known, when the initial state $|\Phi_0\rangle$ is an eigenstate of H_0, the fidelity amplitude $m(t)$,

$$m(t)=\langle\Phi_0|\exp(iHt)\exp(-iH_0t)|\Phi_0\rangle, \tag{6}$$

is just the Fourier transform of the LDOS $\rho_L(E)$, and the form of the LDOS is important in the study of $M(t)$, which is just the survival probability $P(t)$ in this case. However, as shown in Ref. [8], the LDOS cannot explain the perturbation-independent decay of $M(t)$. In fact, the perturbation-independent decay of $M(t)$ appears only for some special set of initial states, that is, it is initial-state dependent. Although a rigorous condition is still lacking, a sufficient condition for such initial states is expressed in a rough way that an initial state should be a narrow wave packet [3]. When the decay of $M(t)$ is initial-state dependent, one should use a quantity that is more general than the LDOS in analyzing the properties of $M(t)$.

The function F_M [the Fourier transform of the fidelity $M(t)$] is a suitable candidate, which is introduced as the follows. Let us first express the fidelity amplitude $m(t)$ in the form

$$m(t)=\int f_m(\epsilon)e^{-i\epsilon t}d\epsilon, \tag{7}$$

where

$$f_m(\epsilon)=\sum_{\alpha,\beta} A_{\alpha\beta}\delta[\epsilon-(E_\alpha-E_\beta)], \tag{8}$$

$$A_{\alpha\beta}=\langle\Phi_0|\beta\rangle\langle\alpha|\Phi_0\rangle C^*_{\beta\alpha}. \tag{9}$$

Note that the quantity $A_{\alpha\beta}$ defined in Eq. (9) is invariant under the transformation

$$|\alpha\rangle\to e^{i\varphi_\alpha}|\alpha\rangle,\quad |\beta\rangle\to e^{i\varphi_\beta}|\beta\rangle,\quad |\Phi_0\rangle\to e^{i\varphi_0}|\Phi_0\rangle, \tag{10}$$

where φ_α, φ_β, and φ_0 are arbitrary phases. That is, it is irrelevant to the relative phases among the states $|\alpha\rangle$, $|\beta\rangle$, and $|\Phi_0\rangle$. The fidelity $M(t)$ can be expressed as

$$M(t)=|m(t)|^2=\int F_M(E)e^{-iEt}dE, \tag{11}$$

where

$$F_M(E)=\int f_m(\epsilon)f_m^*(\epsilon-E)d\epsilon. \tag{12}$$

The quantity $F_M(E)$ is, in fact, the correlation function of $f_m(\epsilon)$. Note that $F_M(E)$ can be calculated directly from properties of the states $|\alpha\rangle$, $|\beta\rangle$, and the initial state $|\Phi_0\rangle$. In the following section, we will investigate numerically the relationship between properties of $M(t)$ and those of $F_M(E)$. To this end, it is convenient to write $F_M(E)$ as

$$F_M(E)=\rho_F(E)e^{i\theta_F(E)}. \tag{13}$$

Some arguments can be given to the reason why the random matrix theory can be used to predict the decaying behavior of the fidelity $M(t)$ in the golden-rule regime, but not in the regime where the perturbation-independent decay of $M(t)$ appears. The initial states, whose fidelities are found to have perturbation-independent exponential decay controlled by the Lyapunov exponent, such as narrow wave packets in the configuration space [3], coherent states [4], momentum eigenstates [9], etc., usually have wide spreading in the chaotic states $|\alpha\rangle$ ($|\beta\rangle$). Such initial states, which are regular in the classical limit, possess certain kind of coherence. It must be pointed out that the semiclassical theory does not predict the perturbation-independent decay of the fidelity for an arbitrary initial state [14]. As shown in the previous paragraph, the function $F_M(E)$, equivalently, $M(t)$, is basically determined by the quantity $A_{\alpha\beta}$ in Eq. (9). In the golden-rule regime, the LDOS $\rho_L(E)$ is narrow, i.e., $C_{\beta\alpha}$ has many small components; as a result, for the quantity $A_{\alpha\beta}$, components of the initial states in the chaotic states, namely $\langle\Phi_0|\beta\rangle\langle\alpha|\Phi_0\rangle$, are not equally effective in the whole (quasi)energy region of H_0 and H. This should suppress the coherence of the initial states, which makes it possible to treat the components of the initial state taking part in the evolution of $M(t)$ effectively as random numbers, like in the random matrix theory treatment in Refs. [4,6,14]. On the other hand, when the perturbation is so strong that the width of the LDOS is comparable to the (quasi)energy bandwidth of H_0, the coherence possessed by the initial states should play a non-negligible role in the evolution of the fidelity. The random matrix theory cannot be applied in this case, since it cannot describe the coherence in the initial states.

III. NUMERICAL STUDY OF FIDELITY AND ITS FOURIER TRANSFORM

A. The model

The unperturbed Hamiltonian H_0 of the kicked top model used in this paper is

$$H_0=\frac{\pi}{2\tau}S_y+\frac{K}{2S}S_z^2\sum_n \delta(t-n\tau), \tag{14}$$

where S is the total angular momentum, τ is the period, and K is a parameter adjusting the strength of the kicks. Without loss of generality, the period τ is set to be unity, $\tau=1$. The model describes a vector spin that undergoes a free precession around the y axis and is periodically perturbed by kicks around the z axis. The time evolution of an initial state $|\Phi_0\rangle$ at $t=0$ is governed by the Floquet operator

$$F_0=\exp\left[-i\frac{K}{2S}S_z^2\right]\exp\left[-i\frac{\pi}{2}S_y\right], \tag{15}$$

where $\hbar$ has been set to be unity. The classical limit of the system, which is obtained by letting $S\to\infty$, with $1/S$ serving as the effective Planck constant, is fully chaotic for $K\gtrsim 9$.

The perturbation κV is chosen in the same way as in Ref. [4], i.e., a slightly delayed periodic rotation of constant angle around the x axis,

$$V=\frac{\pi}{2\tau}S_x\sum_n \delta(t-n\tau-\epsilon). \tag{16}$$

The Floquet operator of the system H is

$$F=\exp\left[-i\kappa\frac{\pi}{2}S_x\right]F_0, \tag{17}$$

which gives the time evolution of an initial state,

$$|\Phi(t)\rangle=F^t|\Phi_0\rangle, \tag{18}$$

where $t=0,1,2,\ldots$. For this model, discussions in Sec. II are still valid, with the eigenstates of H_0 and H changed to those of the Floquet operators F_0 and F, respectively, with the corresponding eigenenergies changed to quasienergies.

In numerical calculations, for the sake of convenience, the perturbation parameter κ is written as

$$\kappa=j\times 10^{-3}. \tag{19}$$

Unless addressed explicitly, the value of S is 500 for the numerical calculations discussed in this paper. Since the time t in the evolution equation (18) takes integer values only, according to Eq. (11), the domain of the variable E in the function $F_M(E)$, which is $[-4\pi,4\pi]$ in the general case, can be reduced to $[-\pi,\pi]$, with the definition of $F_M(E)$ changed accordingly, namely,

$$F_M^{\text{fold}}(E)=\sum_n F_M(E+2n\pi), \tag{20}$$

where n takes the possible values in $(0,\pm 1,\pm 2)$. The function $F_M^{\text{fold}}(E)$ is more closely related to properties of $M(t)$ than the basic one in Eq. (12). It is this function that will be used in what follows in the numerical investigation of the kicked top model and, for brevity, it will be denoted by $F_M(E)$.

Initial states studied in this paper are in most cases coherent states of the SU(2) group [16,17],

$$|z\rangle=Ae^{z^*S_+}|M\rangle, M=-S \tag{21}$$

$$z=-\tan\frac{\theta}{2}e^{i\phi}, \tag{22}$$

where $|M\rangle$ is the eigenvector of S_z, with the eigenvalue M ($M=-S,-S+1,\ldots,S$), and A is the normalization coefficient. The coherence in a coherent state $|z\rangle$ can be easily seen when it is expanded in the states $|M\rangle$. However, in the study of the fidelity, $|z\rangle$ is required to be expanded in the eigenstates of the Floquet operators F_0 and F. Although the expanding coefficients in such chaotic states must contain information on the coherence possessed by the coherent state, manifestation of the information is not so easy. In Sec. III D, we will show numerically that the function $F_M(E)$ introduced in Sec. II can supply some information.

B. Perturbation-independent decay of fidelity and quantum fluctuations

Basic properties of the fidelity $M(t)$ in the kicked top model have been studied in Ref. [4]. Here, as mentioned in the Introduction, we are interested in a perturbation-independent decay of the averaged $M(t)$ with initial coherent states, which decays more slowly than the exponential decay predicted by the semiclassical theory. In order to understand this phenomenon, we study the individual $M(t)$. In the re-

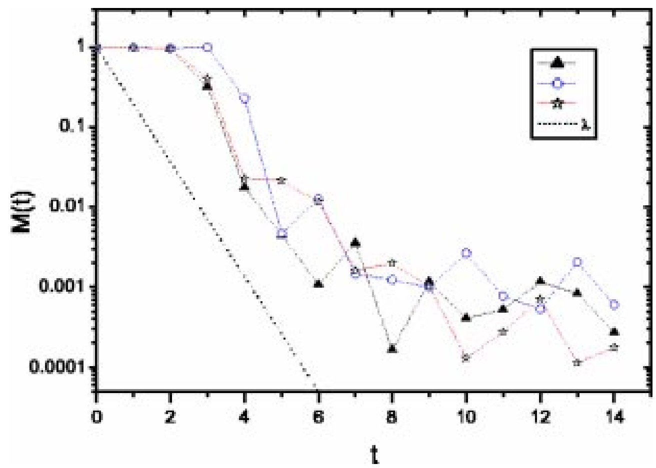

FIG. 1. The fidelity $M(t)$ of three initial coherent states chosen arbitrarily, when $K=13.1$ and $j=6$. The dotted straight line shows the exponential decay with the corresponding Lyapunov exponent $\lambda=1.65$.

gime of the perturbation parameter where the perturbation-independent decay appears, individual $M(t)$ is found to have large fluctuations. See Fig. 1 for some examples of $K=13.1$ and $j=6$, where $M(t)$ are plotted in the logarithm scale. On the other hand, in the golden-rule regime, e.g., $K=13.1$ and $j=1$, no obvious fluctuation in individual $M(t)$ is observed. The large fluctuations are, in fact, quantum effects caused by the coherence in the coherent states. According to the arguments given in Sec. II, they should appear when the expanding coefficients of the states $|\beta\rangle$ in $|\alpha\rangle$ spread in the whole quasienergy band of F_0 effectively, that is, when the half-width of the LDOS becomes comparable to the bandwidth of the quasienergy, which is found numerically for $j\geqslant 3$. In the classical limit $S\to\infty$, the fluctuations should disappear.

As shown in Fig. 1, each $M(t)$ has a "shoulder" in a short initial time, denoted by t_s in what follows, within which it changes slowly. The exponential-type decay of $M(t)$ appears only after t_s. In order to study the average decaying behavior of $M(t)$, their shoulders should be subtracted. To this end, we shift the time variable t to $t_d=t-t_s$ and take average over $M(t_d)$. Here we meet a problem, since individual $M(t)$ has large fluctuations. The averaged $M(t_d)$ may be mainly determined by a small fraction of the $M(t_d)$ taken for averaging, if they have extraordinarily large values. Therefore, in addition to the standard averaged $M(t_d)$, denoted by $M_1(t_d)$,

$$M_1(t_d)=\frac{1}{N}\sum_{i=1}^{N} M(t_d,\Phi_i), \tag{23}$$

we also calculate another averaged $M(t)$, denoted by $M_2(t_d)$, by taking the logarithm of $M(t_d)$ before performing the summation,

$$\ln M_2(t_d)=\frac{1}{N}\sum_{i=1}^{N} \ln M(t_d,\Phi_i). \tag{24}$$

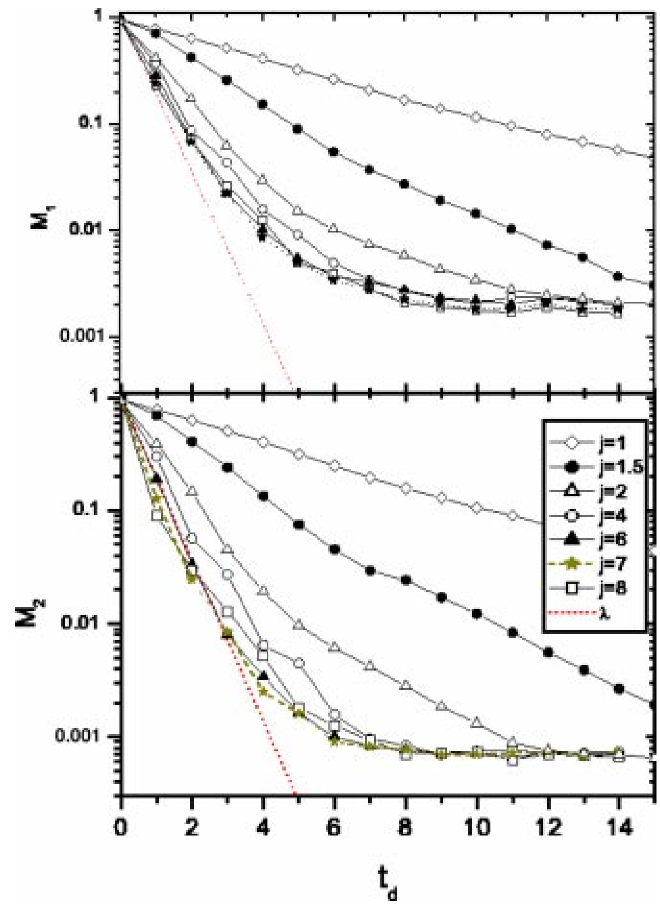

FIG. 2. Decay of averaged $M(t)$ of initial coherent states, from the golden-rule regime ($j=1$) to the perturbation-independent regime, for $K=13.1$. The difference between M_1 and M_2 lies in the averaging procedure [see Eqs. (23) and (24)]. The dotted straight lines represent the exponential decay with the Lyapunov exponent $\lambda=1.65$.

Here $M(t,\Phi_i)$ is the fidelity of an initial coherent state $|\Phi_i\rangle$ defined in Eq. (1), with the dependence on the initial state $|\Phi_i\rangle$ written explicitly.

Numerical results for $M_1(t_d)$ (upper) and $M_2(t_d)$ (bottom) are shown in Fig. 2, with increasing κ (j from 1 to 8), when $K=13.1$. In calculating the two quantities, t_s for each $M(t)$ of $j=1$ and 1.5 was taken to be the first t at which $M(t+1)<0.9$. For $j\geqslant 2$, t_s was determined by the first t at which $M(t)/M(t+1)>1.5$. Only states satisfying $M(t_s)>0.85$ were used in averaging. The reason of taking two methods in calculating t_s is that $M(t)$ decays slowly at $j=1$ and 1.5. The number N of coherent states taken for averaging is 1000. Results in Fig. 2 for $M_1(t_d)$ are in consistence with those in Ref. [4], namely, the decay of $M_1(t_d)$ saturates at an exponential decay that is slower than the one with the Lyapunov exponent. However, when the large fluctuations in individual $M(t)$ are suppressed by the second averaging procedure, Fig. 2 shows that the decay of the averaged fidelity $M_2(t_d)$ saturates at an exponential decay that is close to the Lyapunov decay. The deviation of the saturation decay of $M_1(t_d)$ from the one predicted by the semiclassical theory is caused by the large fluctuations in individual

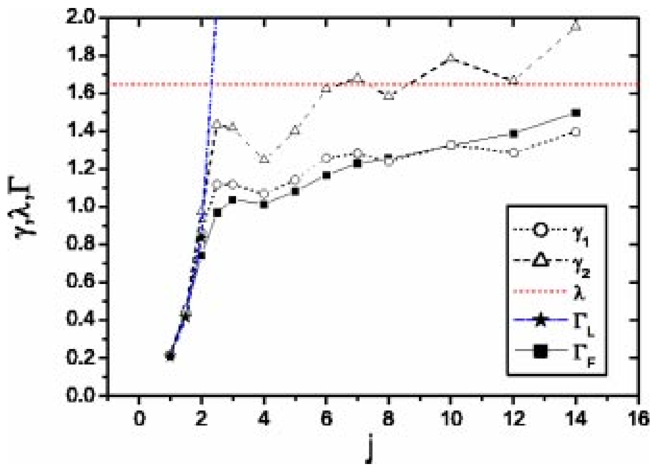

FIG. 3. Values of γ_1 and γ_2 of the exponential decay of $M_1(t_d)$ and $M_2(t_d)$, respectively, at different values of the perturbation parameter, together with Γ_L, the half-width of the averaged LDOS, and Γ_F, half of the averaged half-width of the best fitting Lorentzian form to the central part of $\rho_F(E)=|F_M(E)|$. The value of the Lyapunov exponent of the underlying classical dynamics, $\lambda=1.65$, is indicated by the horizontal dotted line.

$M(t)$. In fact, this can also be seen roughly in Fig. 1.

To show the difference between the decay of $M_1(t_d)$ and that of $M_2(t_d)$ in Fig. 2 in a quantitative way, we calculate their decaying exponents γ_1 and γ_2,

$$M_1(t_d)\propto\exp(-\gamma_1 t_d),\quad M_2(t_d)\propto\exp(-\gamma_2 t_d), \tag{25}$$

the values of which are presented in Fig. 3. Numerically, γ_i ($i=1,2$) are calculated by the best linear fitting to $\ln M_i(t_d)$, with the fitting lines fixed to the values of $\ln M_i(0)$ at $t_d=0$. The points t_d used in fitting are those satisfying $1\geqslant M_2(t_d)\geqslant 0.008$. In Fig. 3, we see that, in the golden-rule regime ($j=1$ and 1.5), γ_1 is close to γ_2 because of the small fluctuation in individual $M(t)$, and both of them are close to the half-width of LDOS, Γ_L, as predicted by the random matrix theory. The difference of the three quantities becomes obvious when $j\geqslant 2$. In the parameter regime $j\geqslant 6$, γ_2 fluctuates around the Lyapunov exponent $\lambda=1.65$, while γ_1 is obviously smaller than λ. The value of γ_2 at $j=14$ is obviously larger than the Lyapunov exponent. This may be due to the reason that the perturbation cannot be regarded as small at this perturbation value. In fact, when the perturbation is so strong that the classical perturbation theory breaks down, $M(t)$ may decay faster than the Lyapunov decay [6]. As discussed above, when S is increased and we are more and more close to the classical limit, the fluctuations in $M(t)$ shown in Fig. 1 should become smaller and smaller, which would make γ_1 approach γ_2. Indeed, we calculate the case of $S=1000$ (κ reduced to half to those of $S=500$), where γ_2 are found to be fluctuating around λ as well and γ_1 a little larger than the values at $S=500$. However, γ_1 of $S=1000$ are still obviously smaller than λ, showing that $S=1000$ is still not in the deep semiclassical regime.

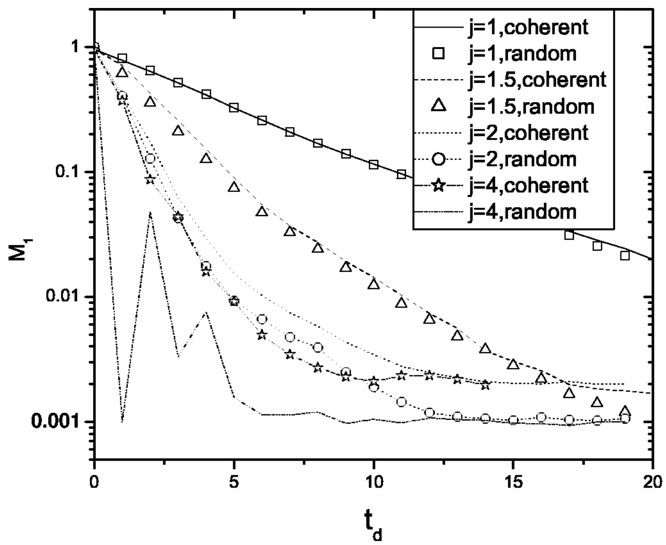

FIG. 4. Comparison of the decay of the averaged fidelity of initial coherent states and that of initial random states in Eq. (27).

C. Dependence of fidelity on initial states

In different regimes of the perturbation parameter, the initial state can play different roles in influencing the behavior of the fidelity. In the golden-rule regime, the LDOS is narrow and, as discussed in Sec. II, the random matrix theory can be used to predict the behavior of the fidelity. In this regime, there should be no difference between the decaying behavior of the fidelity of an initial coherent state and that of an arbitrary initial state, that is, the decay of the fidelity should be initial-state independent. Indeed, in a study of the influence of sub-Planck scale structures [18] on the fidelity in Ref. [14], the averaged fidelity of the initial states,

$$|\Phi_0(T)\rangle=\exp(-iH_0T)|\Phi_0^c\rangle, \tag{26}$$

where $|\Phi_0^c\rangle$ are coherent states, has been found numerically to be independent of the parameter T and almost the same as that of initial coherent states. We compare the averaged fidelity of initial coherent states with that of initial random states of the form

$$|\Phi_0^{\mathrm{random}}\rangle=\sum_\alpha c_\alpha|\alpha\rangle, \tag{27}$$

where c_α are random complex numbers satisfying the normalization condition. It is found that the former ones have shoulders, while the latter ones do not. With the shoulders of the former ones subtracted, the decay of the corresponding averaged fidelity is found to be close to each other in the golden-rule regime. Some of the results are plotted in Fig. 4, where 1000 initial states were used in averaging for each case.

In the perturbation-independent regime of the fidelity of initial coherent states, $M(t)$ of $|\Phi_0(T)\rangle$ is found to be decaying in part of the evolution time in a way similar to that of initial coherent states [14]. This implies that, although the system H_0 is classically chaotic, the time evolution operator

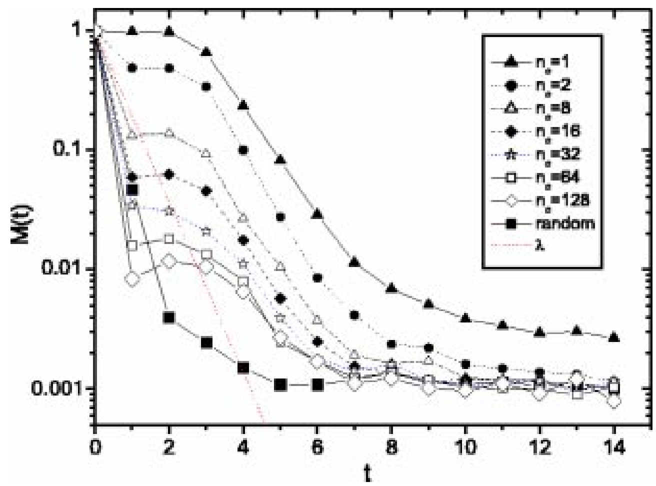

FIG. 5. The averaged fidelity with $|\Phi_0^{\rm rs}(n_c)\rangle$, a random superposition of coherent states, being the initial states. The average is taken in the same way as for $M_1(t_d)$. The solid squares connected by the solid line show the averaged fidelity of initial random states in Eq. (27). The dotted straight line indicates the exponential decay with the Lyapunov exponent λ. $j=6$ for the perturbation parameter.

$\exp(-iH_0T)$ does not destroy the coherence possessed by a coherent state. Indeed, we find that random superpositions of coherent states defined by

$$|\Phi_0^{\rm rs}(n_c)\rangle=\sum_{n=1}^{n_c} e^{i\varphi_n}|z_n\rangle, \qquad (28)$$

where φ_n are random phases and $|z_n\rangle$ are coherent states in Eq. (21), still have some coherence properties of the coherent states. The averaged fidelity with $|\Phi_0^{\rm rs}(n_c)\rangle$ being initial states at the perturbation parameter $j=6$ are presented in Fig. 5. We see that the averaged $M(t)$ with $n_c>1$ has a rapid decrease between $t=0$ and 1, which is faster than the Lyapunov decay for $n_c\geqslant 8$. This indicates that part of the coherence in the coherent components $|z\rangle$ of the initial states $|\Phi_0^{\rm rs}(n_c)\rangle$ is destroyed by the random phases in the initial states. In fact, when $\langle\alpha|\Phi_0\rangle$, the components of initial states in the eigenstates $|\alpha\rangle$ can be treated as random numbers, the random matrix theory analysis predicts that the decay of the averaged $M(t)$ is controlled by the averaged LDOS [14], the half-width of which is larger than the Lyapunov exponent at $j=6$ (Fig. 3).

After the initial transient rapid decrease, the remanent coherence in the initial states, $|\Phi_0^{\rm rs}(n_c)\rangle$, plays a role similar to that of initial coherent states. Indeed, the averaged $M(t)$ changes slowly in some short time intervals after $t=1$, which are analogs of the shoulders of the fidelity of initial coherent states, then, decreases exponentially in a way similar to that of initial coherent states, until they become close to the saturation value, which is about $1/(2S)$. The larger the value of n_c is, the smaller the remanent coherence will be. The fidelity of random superpositions of $|\alpha\rangle$ in Eq. (27) decays faster than the Lyapunov decay shown by the dotted straight line in Fig. 5, as predicted by the random matrix theory.

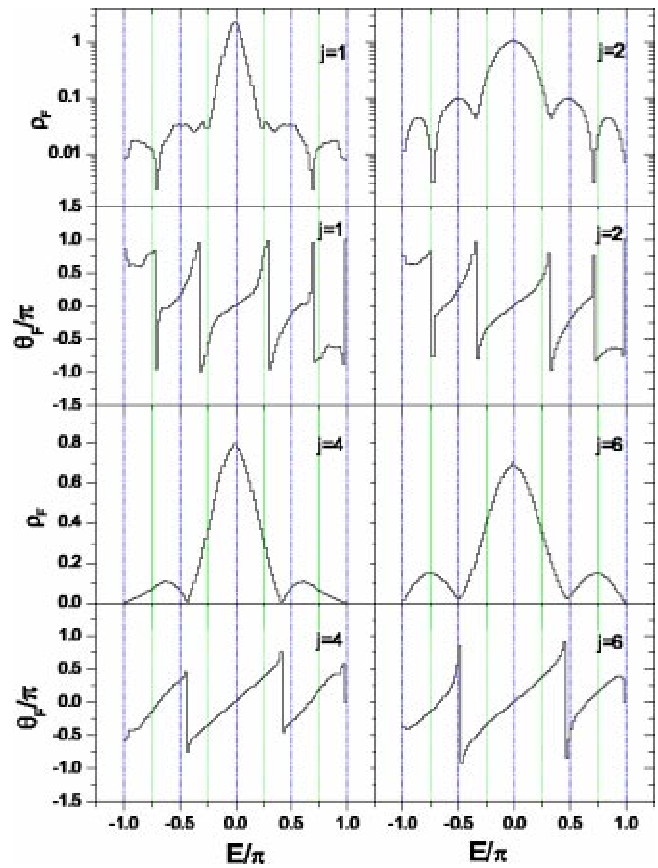

FIG. 6. Typical shapes of $\rho_F(E)$ and $\theta_F(E)$ of the same coherent state, for $j=1$, 2, 3, 4. $\rho_F(E)$ are plotted in the logarithm scale for $j=1$ and 2.

D. Properties of $F_M(E)$ and relation to properties of $M(t)$

As shown in Sec. II, the Fourier transform of $M(t)$, the function $F_M(E)$, is directly determined by the states $|\alpha\rangle$, $|\beta\rangle$, and $|\Phi_0\rangle$. In this section, we study properties of the function $F_M(E)$ in the kicked top model numerically, in particular, its absolute value part $\rho_F(E)$ and its phase part $\theta_F(E)$. Unless addressed explicitly, initial states $|\Phi_0\rangle$ studied in this section are coherent states. It will be shown that properties of the central part of $F_M(E)$ are closely related to the decaying rate and the width of the shoulder of the averaged fidelity $M(t)$.

Typical shapes of $\rho_F(E)$ and $\theta_F(E)$ in cases of different values of the perturbation parameter are shown in Fig. 6, where the same coherent state is used. For $\rho_F(E)$ of $j=1$ and 2, the values of ρ_F are plotted in the logarithm scale. Each $\rho_F(E)$, symmetric with respect to $E=0$, is composed of three or more parts, with a main central part. A manifestation of the coherence in the coherent state $|\Phi_0\rangle$ is that the corresponding phase functions $\theta_F(E)$ have related divisions, as can be seen in Fig. 6. [If $|\Phi_0\rangle$ is taken as a random state $|\Phi_0^{\rm random}\rangle$ in Eq. (27), values of the function $\theta_F(E)$ will be close to zero.]

The central part of each ρ_F is found to be fitted well by the Lorentzian form, with a width denoted by $2\Gamma_F$, to be distinguished from Γ_L for the width of the LDOS. See Fig. 7 for an example of the Lorentzian fit to the central part of the

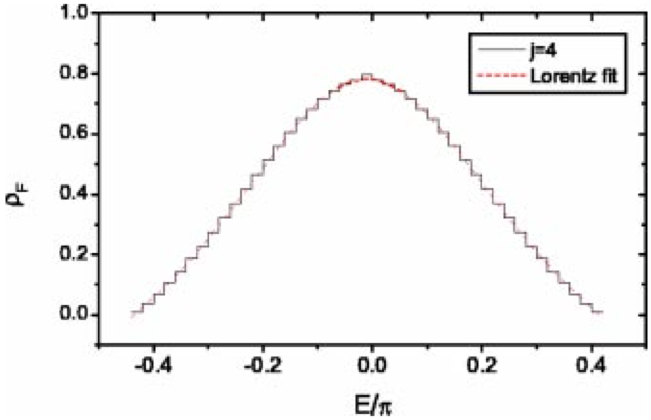

FIG. 7. Lorentzian fit to the central part of the $\rho_F(E)$ in the $j=4$ case in Fig. 6.

ρ_F in Fig. 6 of $j=4$. The Lorentzian form used in the fitting is $\rho_L(E)+a$, where $\rho_L(E)$ is the function on the right-hand side of Eq. (3) and a is a fitting constant. Therefore, the closeness of the central part of $\rho_F(E)$ to the fitting Lorentzian form does not predict directly an approximate exponential decay of $M(t)$ with an exponent Γ_F. In fact, the contribution of the central part of $F_M(E)$ to $M(t)$ in Eq. (11) gives both positive and negative results. The final positive results for $M(t)$ come out, only when the contributions of the other parts of F_M are taken into account. However, the average values of Γ_F, which are shown by solid squares in Fig. 3, are found numerically to be close to the values of γ_1 for $M_1(t_d)$, in the whole parameter regime of j from 1 to 14. When S is taken equal to 1000, the change of the corresponding results of Γ_F are found to be smaller than that of γ_1. Since γ_1 increases only a little, when S is changed from 500 to 1000, it is not clear at the present stage whether or not Γ_F can approach γ_1 in the classical limit.

Numerical results for the phase function $\theta_F(E)$ show that it is approximately linear in the region of E corresponding to the central part of ρ_F (see Fig. 6 for examples),

$$\theta_F(E)\approx k_\theta E, \tag{29}$$

where k_θ is the slope of the approximate linear behavior.

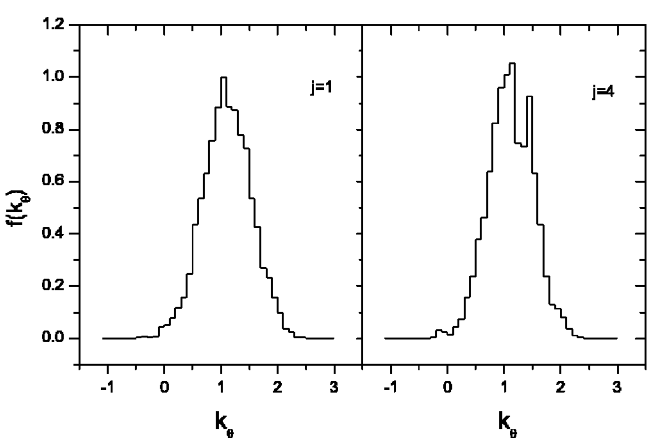

FIG. 8. Distributions of k_θ; the slope of the approximate linear behavior of $\theta_F(E)$ in the central region of $F_M(E)$.

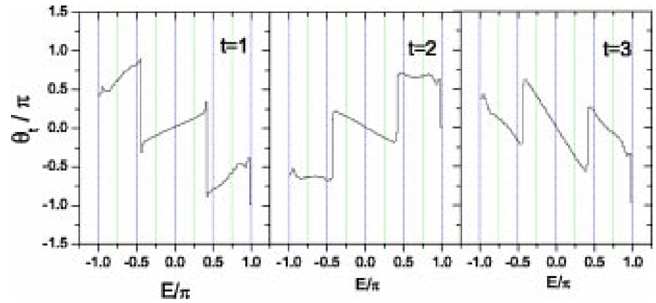

FIG. 9. Change of $\theta_t(E)$ with t, for the $\theta_F(E)$ of $j=4$ in Fig. 6.

Distributions of k_θ are shown in Fig. 8 for some perturbation parameters. The positions of the peaks of the distributions are close to $k_\theta=1$, while the distribution of $j=4$ has another small peak at $k_\theta=1.5$. To illustrate the influence of $\theta_F(E)$ on behaviors of $M(t)$ clearly, let us write $M(t)$ in the form

$$M(t)=\int \rho_F(E)e^{i\theta_t(E)}dE, \tag{30}$$

where $\theta_t(E)=\theta_F(E)-Et$. Some examples of the behavior of $\theta_t(E)$ are shown in Fig. 9. With increasing t, the approximate slope of $\theta_t(E)$ in the central region of $F_M(E)$ changes from positive to negative. For $t\geqslant 2$, the larger the t, the steeper $\theta_t(E)$ will be. As shown below, this feature of $\theta_t(E)$ can be used to estimate the width of the shoulder of $M(t)$.

In order to give an estimation of the shoulder width of $M(t)$, we note that the exponential decay of $M(t)$ should appear when $\theta_t(E)$ is steep enough, specifically, when $\theta_t(-W_F/2)$ is comparable to π, where W_F is the width of the region of E occupied by the main central part of ρ_F. Quantitatively, we use $t_{s\theta}$ to denote the average of the largest t, satisfying

$$|k_\theta - t|\frac{W_F}{2}\leqslant R_c\pi, \tag{31}$$

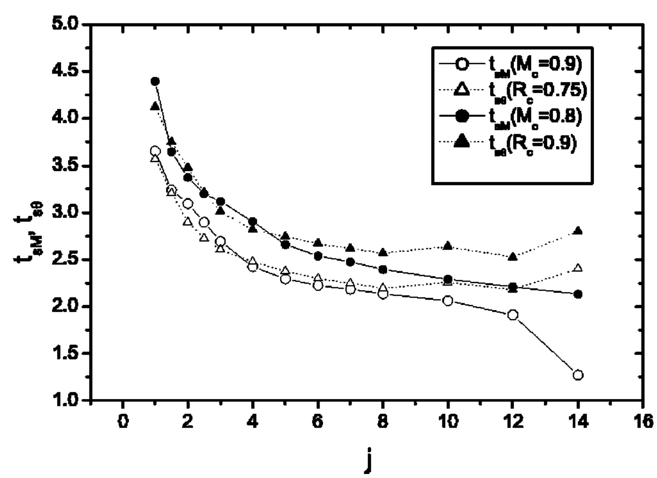

FIG. 10. Two examples of the relationship between $t_{\rm sM}$ and $t_{s\theta}$. They are close to each other, when j is not too large, namely, $1\leqslant j\leqslant 8$.

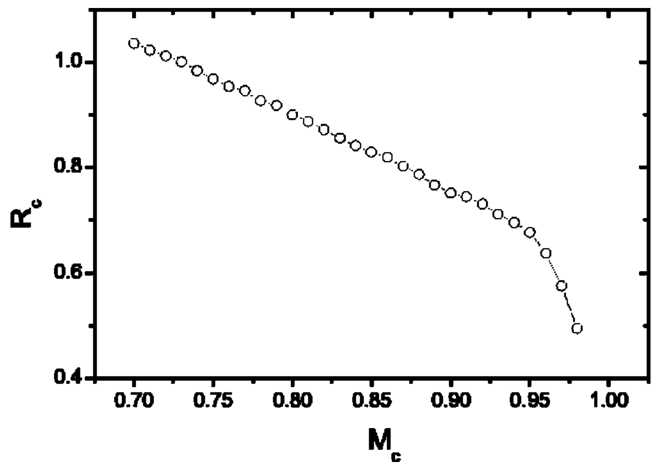

FIG. 11. Dependence of R_c on M_c, required by the smallest value of Eq. (32).

where R_c is a quantity used to show the closeness to π. The average width of the shoulders of $M(t)$, denoted by t_{sM}, are calculated from the averaged $M(t)$ directly. Since t has integer values only in the fidelity $M(t)$ of the kicked top model, we use the linear interpolation in calculating t_{sM} by $M(t_{\mathrm{sM}}) = M_c$, where M_c is a quantity measuring the lower border of the shoulder of the averaged $M(t)$. In studying the relation between t_{sM} and $t_{s\theta}$, for a given value of M_c, the value of R_c is determined by the smallest value of

$$\sum_{j=1}^{8} (t_{s\theta} - t_{sM})^2, \tag{32}$$

in the parameter regime $1 \leqslant j \leqslant 8$. Figure 10 shows two examples of $M_c = 0.9$ with $R_c \simeq 0.75$ and $M_c = 0.8$ with $R_c \simeq 0.9$, where we see that $t_{s\theta} \approx t_{\mathrm{sM}}$ for j between 1 and 8. The difference between $t_{s\theta}$ and t_{sM} becomes increasingly large, when j is larger than 8, which is related to the fact that the perturbation κV is not quite small for $j > 8$. Variation of R_c with respect to M_c is presented in Fig. 11, where R_c is seen to be almost linear with M_c for M_c between 0.7 and 0.95.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we study the crossover of the fidelity from the golden-rule regime to the perturbation-independent decay regime. We show that the deviation of the perturbation-independent decay of the averaged fidelity from the prediction of the semiclassical theory is due to large quantum fluctuations in individual fidelities, caused by the coherence in the initial coherent states. When the quantum fluctuations are suppressed by an appropriate averaging procedure, the perturbation-independent decay of the fidelity is found to be close to the semiclassical prediction.

We also show that when the effective Planck constant is not small enough to guarantee the validity of the semiclassical theory, the crossover does not occur at a point of the perturbation parameter, but takes place in a region of the parameter (Fig. 3). The crossover in this case can neither be analyzed by the semiclassical theory, because the effective Planck constant is not small enough, nor the random matrix theory, since the perturbation-independent decay of the fidelity is initial-state dependent.

Furthermore, a function $F_M(E)$ that is, in fact, the Fourier transform of the fidelity, is introduced to study the fidelity in the crossover region. This function is determined directly by the initial state and the eigenstates of the two Floquet operators concerned. Numerically, for initial coherent states, the absolute value part and the phase part of the function $F_M(E)$ are found to have related division into several subparts, which is a manifestation of the coherence possessed by the initial coherent states. In the whole crossover region, properties of the central part of $F_M(E)$ are found to be closely related to both the exponent of the exponential decay of the averaged fidelity and the length of the short initial time period, within which the fidelity almost does not change due to the coherence possessed by the initial coherent state and the smallness of the perturbation in the classical limit.

The function $F_M(E)$ is studied mainly numerically in this paper. A detailed analytical analysis of the function, e.g., by making use of its expression as the correlation function of $f_m(\epsilon)$ given in Sec. II, would supply further understanding of the behaviors of the fidelity, in analogy with the relation between the LDOS and the survival probability. Information of the coherence in a coherent state may be hidden in its expanding coefficients in the eigenstates of classically chaotic systems. Numerical results in this paper show that the function F_M supplies a useful method of extracting such information. We would like to mention that some other quantities discussed in this paper may serve this purpose as well in other situations, e.g., the quantity $A_{\alpha\beta}$ introduced in Sec. II, which is irrelevant to the arbitrary phases that can be given to the initial coherent state and the quasienergy eigenstates of the two classically chaotic systems.

Note added in proof. After finishing the paper, the authors were made aware of the paper by Silvestrov, Tworzydlo, and Beenakker [19], in which the same definition as Eq. (24) is also introduced and studied in the kicked rotator model.

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